An Active "Cold" Noise Source

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***Abstract*—An active circuit which behaves like a "cold" noise source is described. The circuit, which uses a gallium arsenide FET is given the name COLFET. The appropriate theory is developed and practical circuits described using the circuit. Equivalent noise temperatures of less than 50 K have been measured for a 50-Ω source at 1400 MHz.**

I. Introduction

RECENT DEVELOPMENTS in gallium arsenide FET technology have led to remarkable reductions in both the noise temperature and the capital costs of room-temperature amplifiers.

The theory of the FET and its equivalent circuit is well developed [1] and circuit performance can now be accurately predicted, at least at L-band, by simple parameter measurements.

The input impedance of the FET is predominantly capacitive (there is a small series resistance) with the result that an inductance added to the source circuit is transformed to appear as a noiseless resistor in the gate circuit. This phenomenon allows the development of FET amplifiers with low-input VSWR [3], [4].

Another consequence of the impedance transformation at the input is that only part of the total input resistance contributes noise to the circuit, that portion due to the source inductance being noiseless. This offers the possibility that the input impedance of such an amplifier may be resistive with the apparent temperature of the resistance being much less than ambient and in fact comparable with the noise of the circuit when used as an amplifier.

In this paper the noise theory developed by Pucel *et al.* [1] is revised and the analysis extended to evaluate the performance of the input circuit when used as a noise source. The resultant circuit we have called the COLFET.

II. The FET Amplifier

A. Noise Behavior

The excess noise temperature of the FET amplifier is given as a function of source impedance R_s by Pucel *et al.* [1]

$$T_{\text{FET}} = T_0\left[\frac{R_n}{R_s} + \frac{G_n\omega^2C_{gs}^2}{R_s\cdot g_m^2}(R_s+R_n)^2\right] \tag{1}$$

where C_{gs} is the gate-to-source capacitance of the FET having a transconductance g_m. R_n is the voltage noise generator in the gate circuit,[1] and is equal to the sum of the gate and source resistances external to the channel. G_n is the equivalent current noise conductance of the drain circuit (see Fig. 1) which is associated with the nonthermal channel noise in the FET. The amplifier has a minimum noise temperature $T_{\min}$ given by

$$T_{\min} = T_0\cdot\frac{2\omega C_{gs}}{g_m}\sqrt{R_n\cdot G_n} \tag{2}$$

which occurs for an optimum source impedance $R_{\text{opt}} + jX_{\text{opt}}$ given by

$$R_{\text{opt}} = \left[\frac{g_m^2R_n}{\omega^2C_{gs}^2G_n} + R_n^2\right]^{1/2} \cong \frac{g_m}{\omega C_{gs}}\sqrt{\frac{R_n}{G_n}} \tag{3}$$

and for $X_{\text{opt}} = X_L = -X_{C_{gs}}$. Combining (2) and (3) we obtain

$$\left.\begin{aligned} R_n &= \frac{T_{\min}R_{\text{opt}}}{2T_0} \\ G_n &= \frac{T_{\min}}{R_{\text{opt}}}\cdot\frac{1}{2T_0}\cdot\left\{\frac{g_m}{\omega C_{gs}}\right\}^2 \end{aligned}\right\}. \tag{4}$$

Equations (4) are used to determine R_n and G_n from our measured values of $T_{\min}$ and R_{opt} described here.

B. The Effect of Source Inductance

The effect of added source inductance has been discussed in the literature [2] and [3] and more recently by Williams *et al.* [4] who have used the effect to match an L-band amplifier. The added inductance (L_s in Fig. 1) is transformed to a frequency-independent feedback resistor R_{FB}

$$R_{FB} = \frac{g_m}{C_{gs}S}\cdot L_sS \tag{5}$$

where S is the complex frequency variable. The equivalent circuit for the input is shown in Fig. 2. The impedance transformation of the inductance L_s implied by (5) will be used in the later analysis. An inductance L_s also appears in the input circuit.

[1] We have not considered here the correlation between the drain noise current generator and the induced gate noise generator, here partially included in R_n. However, at the low frequencies, since the induced gate noise varies as ω^2, calculation shows that its contribution is only about 15 percent.

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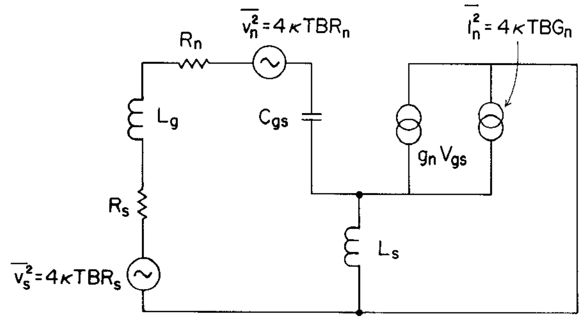

Fig. 1. The full equivalent circuit for the FET used for noise calculations.

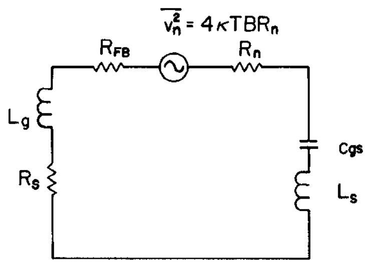

Fig. 2. The equivalent input circuit for the COLFET used in calculating the noise power produced in the source resistance R_s by the gate series noise generator R_n. R_{FB} is the feedback resistance (at $T=0°$) arising from the added FET source inductance which is chosen for a matched input when $R_s = R_{FB} + R_n$. At resonance the optimum source reactance X_L equals the gate-to-source capacitance $X_{C_{gs}}$.

The introduction of lossless negative feedback (as L_s may be considered) has negligible effect on the noise temperature [5].

In practice, the value of L_s is chosen so that $R_s = R_{FB} + R_n$ and the amplifier is thus matched. When R_s is also equal to the optimum source impedance R_{opt}, the amplifier also has minimum noise and thus has a simultaneous gain and noise match. Now since R_{FB} does not change the noise temperature of the amplifier, it must appear to be at zero absolute temperature. This property is used for the COLFET circuit.

III. The COLFET Analysis

The full noise equivalent circuit for the FET is shown in Fig. 1. In considering the noise current flowing in R_s, two contributions arise. The first is due to the noise of the resistance R_n while the second is due to the input noise conductance G_n and its associated generator. These two contributions are separately calculated and combined in the final computation of an equivalent temperature for the total input resistance of the circuit.

A. The Input Noise Generator

In Fig. 2, an equivalent circuit for the input section of Fig. 1 is given.

The resistance R_{FB} has been seen to arise due to the impedance transformation of the inductance L_s by the device. L_g is an inductance added to tune out C_{gs} and L_s so that the input impedance of the circuit becomes $R_{FB} + R_n$.

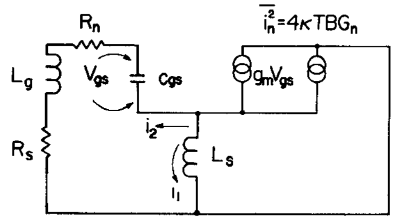

Fig. 3. The equivalent circuit of the COLFET used in calculating the noise power produced in the source resistance R_s by the drain current generator G_n. Here, L_s is the source inductor added to the FET, transconductance g_m. There is a division of current from the G_n generator into path i_1 through L_s, and i_2 through the loop including R_s.

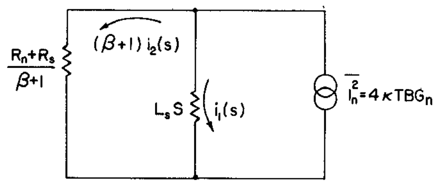

Fig. 4. Transformed equivalent circuit derived from Fig. 3 for use in the calculation of the contribution of the output-noise generator to the noise in the input circuit.

The match condition gives

$$R_s = R_{FB} + R_n. \tag{6}$$

The noise current due to the R_n generator in this circuit is given by

$$\overline{i_1^2} = \frac{\overline{v_n^2}}{(R_s + R_{FB} + R_n)^2} \tag{7}$$

and for input matching

$$\overline{i_1^2} = \frac{\overline{v_n^2}}{4R_s^2}. \tag{8}$$

B. The Output-Noise Generator

The circuit of Fig. 3 is used to derive the noise current in R_s due to the output-noise generator.

If we now introduce a parameter β where

$$\beta(S) = \frac{g_m}{C_{gs} S} \tag{9}$$

β is recognized as a complex gate-to-drain current gain. We can now transform Fig. 3 to give Fig. 4. In so doing, L_g and C_{gs} have been removed as they are almost series resonant. Now writing

$$i_N(S) = \sqrt{\overline{i_n^2}(s)} \tag{10}$$

$$i_2(S) = \frac{i_N(S)}{\beta+1} \cdot \frac{L_s S}{\dfrac{R_n + R_s}{\beta+1} + L_s S} \tag{11}$$

$$= \frac{i_N(S)}{\beta+1} \cdot \frac{(\beta+1) L_s S}{R_n + R_s + (\beta+1) L_s S} \tag{12}$$

and hence

$$\overline{i_2^2} = \frac{\overline{i_n^2}}{|\beta|^2+1}\cdot\frac{R_{FB}^2}{(R_n+R_s+R_{FB})^2}. \tag{13}$$

Using (6) and recognizing that

$$|\beta| = g_m X_{C_{gs}} \tag{14}$$

$$\overline{i_2^2} = \overline{i_n^2}\,\frac{(R_s-R_n)^2}{4R_s^2}\cdot\frac{1}{g_m^2 X_{C_{gs}}^2+1}. \tag{15}$$

C. The Total Noise at the Input

The equivalent noise temperature contribution for a noise current specified by $\overline{i^2}$ flowing in R_s is given by

$$\overline{i^2}\cdot R_s = \kappa T_{eq}\Delta f \tag{16}$$

from which

$$T_{eq} = \overline{i^2}\cdot\frac{R_s}{\kappa\Delta f}. \tag{17}$$

Combining the contributions of the input-circuit and output-circuit noise generators and simplifying we obtain

$$T_{eq} = T\left\{\frac{G_n}{(g_m^2X_c^2+1)}\cdot\frac{(R_s-R_n)^2}{R_s}+\frac{R_n}{R_s}\right\}. \tag{18}$$

Differentiating with respect to R_s we obtain the value of R_s for minimum T_{eq}

$$R_{s_{min}}^2 = R_n^2 + \frac{R_n}{G_n}(g_m^2X_c^2+1) \tag{19}$$

since

$$\frac{1}{G_n} \gg R_n$$

and

$$g_m^2X_c^2 \gg 1$$

$$R_{s_{min}} \approx \frac{g_m}{\omega C_{gs}}\sqrt{\frac{R_n}{G_n}}. \tag{20}$$

This result is the same as obtained for the amplifier noise analysis.

A further general relationship is derived for the optimum source impedance R_{opt}, by substituting (3) into (1), to obtain the minimum noise temperature T_{min}

$$\frac{T_{min}}{T_0} = \frac{2R_{gn}}{(R_{opt}-R_{gn})}. \tag{21}$$

Likewise, substituting (19) into (18) (assuming $g_m^2X_c^2 \gg 1$) we obtain for the minimum value of $T_{eq,min}$

$$\frac{T_{eq,min}}{T_0} = \frac{2R_{gn}}{(R_{opt}+R_{gn})}. \tag{22}$$

Taking the quotient of these relations (21) and (22) we obtain

$$\frac{T_{eq,min}}{T_{min}} = \frac{R_{opt}-R_{gn}}{R_{opt}+R_{gn}} \tag{23}$$

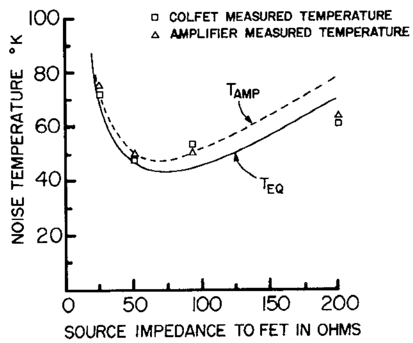

Fig. 5. The measured noise temperature of the COLFET input and also its noise temperature as an amplifier, both plotted as a function of the source resistance R_s. The solid curve T_{eq} is the equivalent temperature of the COLFET calculated from theory developed in the text. The dashed curve T_{AMP} is based on the Pucel theory of the noise temperature of the FET amplifier.

where $T_{eq,min}/T_{min}$ is the ratio of the minima on our theoretical curves of Fig. 5.

IV. Measurements

All the measurements described here were made using a low noise receiver, itself having an FET front end. A Maury Microwave LN_2 load was used in the noise temperature measurements together with an AIL precision attenuator for the Y-factor method.

A. Results for the Amplifier

We have found the values of R_{opt} and T_{min} experimentally by measuring the amplifier noise temperature for various values of source impedance seen by the FET. A series of different quarter-wave transformers were installed in the amplifier between the FET and the source. The measured values of amplifier noise temperature are plotted in Fig. 5 against the various values of source impedances used. From these data we obtain values of $T_{min} = 48°$ and $R_{opt} = 70\ \Omega$ at 1.4 GHz. Using the known values for the Mitsubishi 1402 of $C_{gs} = 0.55$ pF, $g_m = 0.025$ (at 1.5 V and 7 mA drain current) we obtain from (4) the values for the noise generators

$$R_n = 5.8\ \Omega$$
$$G_n = 0.0296\ \mho.$$

These values are used to calculate the T_{AMP} curve plotted in Fig. 5 from (1) and later used in the calculation of the COLFET input noise temperature.

B. Results for the COLFET

Two sets of measurements were made on the COLFET circuit. The first of these (shown in Fig. 5) is the equivalent temperature T_{eq} presented by the circuit when optimized to provide different input impedances. As in the amplifier case, the lowest noise occurs at about 70 Ω. The agreement with the theory is particularly good. The calculated noise at high impedances (200 Ω) is sensitive to the values taken for C_{gs} while the lower impedance region is sensitive to R_n.

The configuration for the second set of measurements of the COLFET to determine the frequency behavior is shown

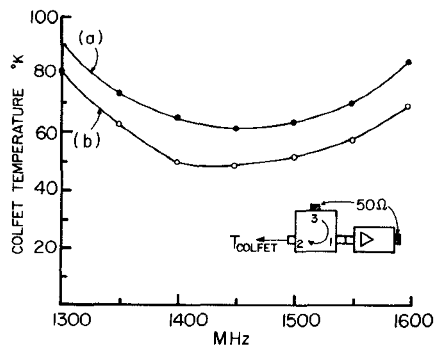

Fig. 6. The noise temperature of the COLFET cold load plotted as a function of frequency measured (a) with a circulator and (b) without. Curve (b) was shifted 50 MHz to the right with the circulator addition (see text).

in Fig. 6. The COLFET is connected to the receiver input in turn with the calibrating LN_2 and T_0 loads. Measurements were made with and without the circulator. The results are presented in Fig. 6(a) and (b). A small detuning effect of about 50 MHz occurs when the COLFET is attached to the circulator port, and this has been taken into account in making plot (b).

In practice, although the COLFET can be used by itself as a cold load, it is reactive out of band. Consequently, the use of the circulator provides a matched cold load with only a small increase in cold load temperature due to the insertion loss of ports 1→2.

V. Applications of the COLFET

Some immediate applications for this technique are as a cold load, a circulator termination, or as a reference load in a Dicke radiometer.

A. Cold Load

The COLFET is used as a conventional cold load, for noise temperature measurement of low-noise receivers. For the application, the COLFET is calibrated against a liquid nitrogen load. The COLFET is found to be stable with time at constant bias conditions, but its temperature will be proportional to ambient changes. Because of the small size, temperature control can be simply provided.

B. Circulator Termination

The COLFET is used as a cold terminating port on a 3-port junction circulator. In this application, the noise arising from the third port of a parametric amplifier circulator and which is reflected from a mismatched source such as an antenna feed, can be reduced in the ratio of the cold load temperature to ambient temperature. This reduces the system noise contribution from the antenna mismatch effect. In the same way, noise radiated from a circulator ahead of a low-noise FET amplifier can be similarly reduced.

C. Radiometer Reference Load

The COLFET is used as a cold load in a Dicke-type radiometer in Radioastronomy applications. The COLFET provides a comparison load of approximately the same temperature as the antenna of 50 K in this frequency range.

VI. Conclusions

In the short time that has been available for evaluation of the COLFET it has proved to be an extremely useful circuit. Its compactness will appeal to those who have carried a dewar of liquid nitrogen to inaccessible places to calibrate equipment or set up experiments and we expect it to find wide application.

Acknowledgment

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